

# Bulk waves and dynamical behaviour in elastic solids reinforced by two families of strong fibres

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**Abstract** In the study of wave propagation, bulk waves exist in infinite homogeneous bodies and propagate indefinitely without being interrupted by boundaries or interfaces. Such waves may be decomposed into finite plane waves propagating along arbitrary directions of the solid. The properties of these waves are determined by the dependence between the propagation direction and the constitutive properties of media. Three types of such waves may be distinguished in connection to the three displacement vectors which determine an acoustic polarization. These three polarization vectors are mutually orthogonal, but in most cases they are neither perpendicular nor parallel to the propagation direction. There are a number of papers and books describing wave front surfaces as a consequence of the study of the acoustic tensor, which, in fact, represents the propagation condition of such waves. The surfaces associated with wave front surfaces are slowness surfaces, with slowness defined by the inverse of the wave front speeds, and energy flow surfaces, also known as group velocity surfaces. Here slowness surfaces are studied analytically and numerically to obtain information about wave propagation in arbitrary directions. The degree of wave surface deviation depends upon degrees of anisotropy, and may give valuable information about dynamic deformations. The materials, used in the present analysis, are fibre-reinforced materials with two families of continuous elastic fibres. Since fibres are much stronger than a matrix, anisotropic properties are emphasized. Coordinate-free constitutive relations give the possibility of considering fibres embedded in such materials as extensible or inextensible. Such an approach gives the opportunity to follow behaviour directly considering both the geometry and the stiffness of the fibres. The material considered here is reinforced by two families of mechanically equivalent fibres and, therefore, behaves like orthotropic, having axes of symmetry along bisectors of the fibre directions and along the normal to the plane tangent to the fibres. Such a material has nine independent material constants. When constraints of inextensibility are imposed on the fibres, it leads to the so-called ideal fibre-reinforced material, which may be obtained from materials with extensible fibres in a limiting process. In order to follow the influence of fibre direction on the mechanical properties, constitutive relations are modelled as a function of the fibre directions in each point of the continuous media. An often-used representation of such materials is an epoxy resin–carbon fibre composite whose material constants, for materials reinforced by one family of fibres, are determined using ultrasound methods. Here we consider the dynamical behaviour of the material reinforced by two families of

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